Bunching of Satellites in Their Orbital Plane

RAJENDRA C. NIGAM*
System Sciences Corporation, Falls Church, Va.

BY deploying two or more satellites in one launch, we can make maximum utilization of the launch capacity of our boosters at the minimum cost. However, some limiting restrictions are imposed on the orbits of the individual satellites. Satellites deployed by spring mechanism or vernier thrust normally all lie in one orbital plane and are bunched together initially. It is usually desirable to design a satellite configuration that will minimize bunching of two or more satellites in order to provide better coverage and reduce vulnerability to electronic jamming.

This problem has been treated by Rinehart and Robbins.[1] It is not intended here to deal with this problem in all its details nor to choose the optimum procedures to minimize bunching. The orbital aspects of deployment of satellites has been dealt with in great detail by the author.[2] This note is confined to summarizing those supplementary equations dealing specifically with spatial and angular bunching in orbits that are almost circular and to showing the consequences of a particular mode of deployment on the bunching of satellites.

An error in injection velocity represented by ΔV is related to the errors Δa, ΔP, and Δe by the following equations:

$$\Delta a/a = -2\Delta V/V \quad (1)$$

$$\Delta P/P = -3\Delta V/V \quad (2)$$

$$\Delta e = [1 - Q(2 - Q)\cos^2\beta]^{\frac{1}{2}} \quad (3)$$

where $Q \equiv (V_i/V_c)^2$, V_i is the velocity of injection, V_c is the circular velocity, β is the elevation angle of injection, and a, P, and e are standard orbital elements. Representing the direction along the orbit by the Y axis and the direction along the radius vector pointing outward by the Z axis, the motion of a satellite relative to another satellite in circular orbit around a spherical earth is given by Jensen, Townsend, Kraft and Kork[3] as follows:

$$Y = 4\dot{Y}_0 \sin\omega t/\omega - 3\dot{Y}_0 t \quad (4)$$

$$Z = 2\dot{Y}_0(\cos\omega t - 1)/\omega \quad (5)$$

where the satellite has been injected from the origin along the Y axis with a velocity $\dot{Y}_0$, and ω is the angular velocity of the satellite. If the satellite is injected from the origin along the Z axis with a velocity $\dot{Z}_0$, then the motion is given by

$$Y = 2\dot{Z}_0(1 - \cos\omega t)/\omega \quad (6)$$

$$Z = \dot{Z}_0 \sin\omega t/\omega \quad (7)$$

Suppose that the injecting vehicle is in a 6-hr circular orbit moving at 16,000 fps at a geocentric radius of 9052 naut miles. It deploys 7 satellites at intervals of 10 sec with a 5 fps differential ΔV between consecutive satellites. For $\Delta V = 5$ fps in the direction of the local horizon, the differentials of the orbital elements will be as follows: $\Delta P = 0.34$ min, $\Delta a = 5.66$ naut miles, and $\Delta e = 0.000625$.

The previous results show that the maximum radial distance between two consecutive satellites at their apogees is about 11 naut miles. The launch vehicle travels 158 naut miles (equivalent to 1° along the arc) within 1 min in its orbit. But when the seventh satellite has been deployed, the first one is only 900 ft behind it [$\Delta S = \frac{1}{2}at^2 = \frac{1}{2}(\frac{1}{2})(6 \times 10)^2 = 900$ ft]. This amounts to an angular separation of 3 sec of arc between the first and the last satellite at the moment the last satellite has been deployed. The reaction on the injecting vehicle is assumed to be negligible.

Now let us compute the radial separation of two consecutive satellites at the moment of their deployment. The second satellite has a velocity of 16,005 fps in the direction of the local horizon, which, in effect, gives the orbit an eccentricity of 0.000625. In 10 sec the satellite has already moved about 0°.17 in the elliptic orbit starting from the injection point, which is the perigee. The radial separation between the first two satellites is then given by

$$\Delta r = a\{[(1 + e)/(1 + e\cos v)] - 1\} \quad (8)$$

Putting $a = 9052$ naut miles, $e = 0.000625$, and $v = 11$ min of arc, we get $r = 0.3$ ft. The radial separation between the first satellite and the seventh is about 5 ft at the moment of deployment of the seventh satellite.

The previous results show that serious bunching exists at the moment of deployment of the configuration following the assumed mode of injection. In spite of this, the faster moving satellite is not as close to the slower moving satellite when being overtaken by the latter (a well-known paradox) as one might intuitively expect. At the moment of overtaking, $Y = 0$. Therefore, from Eqs. (4) and (5) and using

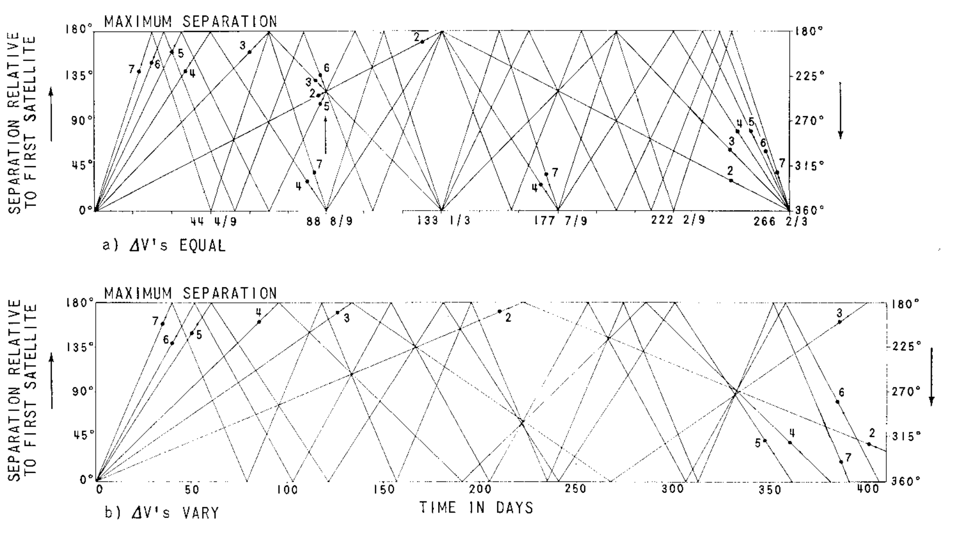

Fig. 1 Relative separations of satellites 2–7 with respect to the first as a function of time, when differential velocities are a) equal and b) unequal.

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* Member, Technical Staff; now with Bellcom, Inc., Washington, D. C. Member AIAA.

Table 1 Comparison of bunching relative to first satellite for a group of satellites launched with equal ΔV's vs a group launched with unequal ΔV's

Satellite, n	$V_n - V_{n-1}$, fps	Drift rate, deg/day	Times of bunching after injection, days
2	5	1.35	266.67
3	5	2.70	133.33
4	5	4.05	88.89
5	5	5.40	66.67
6	5	6.75	53.33
7	5	8.10	44.44
2	3	0.81	444.44
3	2	1.35	266.67
4	2	1.89	190.48
5	4	2.97	121.21
6	2	3.51	102.56
7	4	4.59	78.43

ω = 0.000291 rad/sec and $\sin\omega t = \omega t - (\omega t)^3/6$, we get $Z \simeq 3.4$ naut miles at 1.22 hr after injection.

The relative drift rate of one satellite with respect to another satellite moving with period P (in hr) is 8640 $(\Delta P/P^2)°$/day or 1080 $(\Delta V/V_c)°$/period, where ΔP is the difference in period.

The relative drift rate of two consecutive satellites is 1°.35 per day. The last satellite will therefore be drifting at 8°.1 per day from the first one. Thus, after its first revolution, the earliest duration for one satellite to be overtaken by another satellite is 44.44 days, and the first satellite overtakes the last one when they are 280° in their orbit from the injection point. After 266.67 days at an angular distance of 240° from the injection point, each one of the seven satellites is being overtaken by one another. Thus, every 266.67 days all of the satellites bunch together in the same direction and are radially separated by 8 naut miles each. The results are summarized in the first group in Table 1 and in Fig. 1a. Note that the maximum separation of any satellite relative to another cannot exceed 180°, and hence the top horizontal lines in Figs. 1a and 1b represent maximum separation and the bottom horizontal lines represent bunching. The times of mutual bunching between other satellites can easily be read from the figure. As for example, satellites 2, 3, 5, and 6 bunch together among themselves at every 88.89 days as seen from Fig. 1a.

The bunching can be minimized by choosing random differential velocities; for example, let us deploy 7 satellites with $V_n - V_{n-1}$ = 3, 5, 7, 11, 13, and 17 fps, respectively. The results are shown by the lower group in Table 1 and in Fig. 1b. Bunching is greatly reduced, and the total increment in velocities is now only 56 fps, as compared to 105 fps for the previous case.

Thus, if random differential velocities are properly chosen, we can reduce bunching to the minimum and at the same time ensure a suitable distribution of the satellites in their orbit to provide the desired coverage for communication links spread throughout the globe.

References

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[2] Nigam, R. C., "Orbital aspects of deployment of satellites," J. Astronaut. Sci. **XI,** 37–45 (1964).

[3] Jensen, J., Townsend, G. E., Kraft, J. D., and Kork, J. D., *Design Guide to Orbital Flight* (McGraw-Hill Book Co., Inc., New York, 1962), p. 156.

A "Spark-Plug" Starter for Arc Plasma Generators

T. A. Barr Jr.* and R. F. Mayo†
U. S. Army Missile Command, Redstone Arsenal, Ala.

Accurately timed and consistent, simultaneous starting of two or more arc-type plasma generators[1] was required for plasma experiments under a wide range of ambient pressures, some as low as 10^{-4} torr, and open-circuit voltages as low as 200 v d.c. with an arc gap of $\frac{3}{8}$ to $\frac{5}{8}$ in. A high-frequency voltage across the electrodes was satisfactory except that insulation failure resulted. Squibs and shorting links were considered to be unsatisfactory because of the many problems associated with their use. Figure 1a shows an arc starter, which meets the above requirements. A high voltage between the spark-plug tip and the grounded seat causes a transient, high-current arc to form in the channel in the plastic insert. Material vaporized from the channel wall is heated in the arc and ejected as a tongue of plasma into the electrode gap of the plasma generator. The conductivity of the ejected gas is high enough to initiate the arc in the plasma generator. The only modification to the plug is the removal of the ground electrode; any type with a protruding center insulator but without a built-in resistor will do. The plastic insert is made of any one of several plastics (e.g., Lucite, polyethylene, or the fluorocarbon polymers), which do not shatter under the electrical or other stresses developed in the starter. The insert is essential for consistent operation of the starter. The replaceable seat is a washer made of copper, stainless steel, or other noncorroding metal; it is not absolutely necessary but is convenient and prevents wear on the permanent housing.

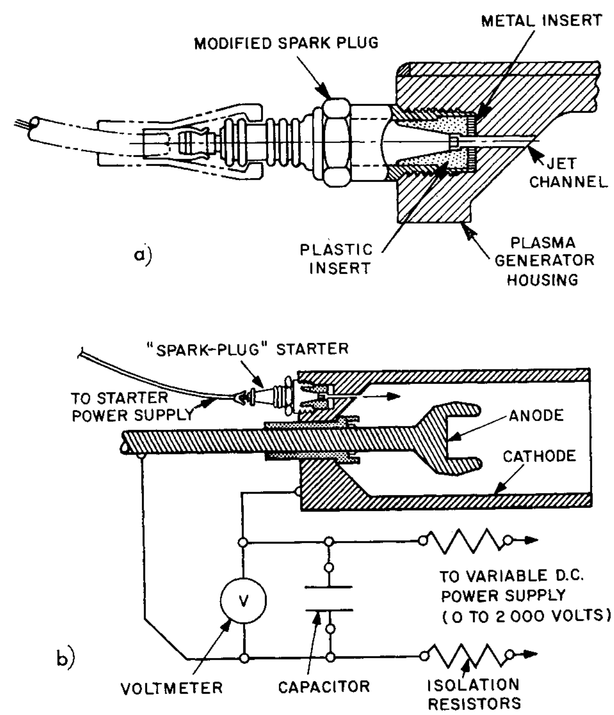

Fig. 1 a) Cross-sectional diagram of an arc starter; b) circuitry for performance characteristics experiment; arc ignition occurs near the point of minimum separation between anode and cathode.

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* Chief, Plasma Physics Branch, Physical Sciences Laboratory, Directorate Research and Development. Member AIAA.

† Research Physicst, Plasma Physics Branch, Physical Sciences Laboratory, Directorate Research and Development.